About Some Distinguishing Features of the Weak Interaction

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Abstract

In this work it is shown that, in contrast to the strong and electromagnetic theories, additive conserved numbers (such as lepton, aromatic and another numbers) and γ_5 anomaly do not appear in the standard weak interaction theory. It means that in this interaction the additive numbers cannot be conserved. These results are the consequence of specific character of the weak interaction: the right components of spinors do not participate in this interaction. The schemes of violation of the aromatic and lepton numbers were considered.

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1. Introduction

Strong and electromagnetic interactions theories are left-right systemic theories (i.e. all components of the spinors participate in these interactions symmetrically). In contrast to this only the left components of fermions participate in the weak interaction. This work is dedicated to some consequences deduced from this specific feature of the weak interaction.

2. Distinguishing Features of Weak Interactions

As it is well known from the Neuter theorem [1], conserving currents appear at global and local abelian and nonabelian gauge transformations.

These values for local gauge transformations are:

electromagnetic current–j^μ

$$j^\mu = e\bar{\Psi}\gamma^\mu\Psi, \tag{1}$$

where e is an electrical charge;
the current of strong interactions–$j^{a\mu}$

$$j^{a\mu} = q\bar{\Psi}T^a\gamma^\mu\Psi, \tag{2}$$

where q is charge of a strong interactions, T^a is $SU(3)$ matrix, a is color.

The currents S_i^μ obtained from the global abelian transformation is

$$S_i^\mu = i(\bar{\Psi}_i\partial_\mu\Psi_i), \tag{3}$$

(where i characterizes the type of the gauge transformation) and the corresponding conserving current (the forth component of S_i^μ) is

$$I_i = \int S_i^0 d^3x = \int \epsilon\bar{\Psi}_i\Psi_i d^3x, \tag{4}$$

where ϵ is the energy of fermion Ψ_i.

Then conserving values of global gauge transformations are:
electrical number Q

$$Q = \int e\epsilon\bar{\Psi}\Psi d^3x; \tag{5}$$

baryon numbers B

$$B = \int \epsilon\bar{\Psi}_B\Psi_B d^3x; \tag{6}$$

the lepton numbers $l_i (i = e, \mu, \tau)$

$$l_i = \int \epsilon\bar{\Psi}_{l_i}\Psi_{l_i} d^3x; \tag{7}$$

aromatic numbers and etc.

In the vector (electromagnetic and strong interactions) theories all components of spinors (Ψ_L, Ψ_R) participate in interactions. In contrast to the strong and electromagnetic theories, the right components of the spinors $(\bar{\Psi}_R, \Psi_R)$ do not participate in the weak interaction, i. e. this interaction does not refer to the chiral theory (in the chiral theory the left and right components of fermions participate in the interaction in the independent manner). Such character of the weak interaction leads to certain consequences: impossibility to generate fermion masses [2] and to the problem of jointing this interaction to the strong and electromagnetic interactions [3].

Let us consider another consequences of this specific feature of the weak interaction.

The local conserving current $j^{\mu i}$ of the weak interaction has the following form:

$$j^{\mu i} = \bar{\Psi}_L\tau^i\gamma^\mu\Psi_L, \tag{8}$$

where $\bar{\Psi}_L, \Psi_L$ are lepton or quark doublets

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_{iL}$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_{iL}, \qquad i = 1 - 3.$$

If now we take into account that in the right components of fermions $\bar{\Psi}_{iR}, \Psi_{iR}$ do not participate in the weak interaction, then from (4) for abelian currents we get

$$I_i = \int \epsilon \bar{\Psi}_{iL} \Psi_{iL} d^3x \equiv 0, \tag{9}$$

i.e. (in contrast to the strong and electromagnetic interactions) no conserving additive numbers appear in the weak interaction.

It is clear that the lepton and aromatic numbers appear outside the weak interaction and it is obvious that the interaction, where these numbers appear, must be a left-right symmetric one.

It is also clear that, since in the weak interaction no conserving additive numbers appear, then the additive (aromatic, lepton and etc.) numbers can be violated in the weak interaction . Thus, the violation scheme of aromatic numbers, as well known, is Cabibbo-Kobayashi-Maskawa matrices [4, 5]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{10}$$

where u, d, s, c, b, t are quarks.

The analogous scheme can be used to discribe of the violation of the lepton numbers [5]

$$V = \begin{pmatrix} X_{ee} & X_{e\mu} & X_{e\tau} \\ X_{\mu e} & X_{\mu\mu} & X_{\mu\tau} \\ X_{\tau e} & X_{\tau\mu} & X_{\tau\tau} \end{pmatrix}, \tag{11}$$

where e, μ, τ are leptons.

It is necessary to stress that, probably, in the weak interaction there is no conserving baryon number B, third projection the weak isospin (I_3^w), and etc. (see Eqs (7),(9)), but it leads to any consequences since the local electric, strong and weak currents are conserved.

All the above considered violations of these numbers in the weak interaction are the direct violations.

Now we consider the problem: Can the γ_5 anomaly appear in the weak interaction?

For this purpose we, at first, use the functional-integral measure method for the vector theory, considered by K. Fujikawa [6], and then use this method for the weak interaction.

At the γ_5 transformations

$$\Psi(x)' \to exp(i\alpha(x)\gamma_5)\Psi$$

$$\bar{\Psi}(x)' \to exp(i\alpha(x)\gamma_5)\bar{\Psi}, \tag{12}$$

we get the following increment to lagrangian $\mathcal{L}$

$$\mathcal{L} \to \mathcal{L} - \partial_\mu \alpha(x)\bar{\Psi}\gamma^\mu\gamma_5\Psi - 2mi\alpha(x)\bar{\Psi}\gamma_5\Psi, \tag{13}$$

where

$$\mathcal{L} = \bar{\Psi}(i\hat{D} - m)\Psi + (\frac{g^2}{2})trF^{\mu\nu}F_{\mu\nu}$$

and $\alpha(x)$ is infinitesimal parameter.

Functional- integral measure is defined by the following equation:

$$d\mu \equiv \prod_x DA_\mu(x)D\bar{\Psi}(x)D\Psi(x) \tag{14}$$

Under infinitesimal transformations this measure does not remain invariant and we get (see Appendix and work [6])

$$d\mu' \to d\mu exp[i\int \alpha(x)(\frac{1}{8\pi^2})tr^*F^{\mu\nu}F_{\mu\nu}dx], \tag{15}$$

where ${}^*F^{\mu\nu} = \epsilon^{\sigma\rho\mu\nu}F_{\sigma\rho}$.

According to the requirement of the measure invariance at this infinitesimal transformation, we obtain

$$\partial_\mu(\bar{\Psi}\gamma^\mu\gamma_5\Psi) = 2mi\bar{\Psi}\gamma_5\Psi - i\frac{1}{8\pi^2}\epsilon^{\sigma\rho\mu\nu}F_{\sigma\rho}F_{\mu\nu} \tag{16}$$

The second term of the right part of (16) is the γ_5 anomaly term.

In the case of the weak interaction $\Psi_R = \bar{\Psi}_R \equiv 0$ and

$$\bar{\Psi} \to \Psi_L, \Psi \to \Psi_L, \tag{17}$$

then the functional-integral measure is zero (see Appendix)

$$d\mu \equiv 0, \tag{18}$$

and the γ_5 anomaly term of the right part of Eq. (16) is also zero.

So, we see that in the weak interaction the γ_5 anomaly does not appear and the equation type of Eq.(16) for the weak interaction has the following form:

$$\partial_\mu(\bar{\Psi}_L\gamma^\mu\gamma_5\Psi_L) \equiv 0. \tag{19}$$

3. Conclusion

In this work it was shown that, in contrast to the strong and electromagnetic theories, additive conserved numbers (such as lepton, aromatic and another numbers) and γ_5 anomaly do not appear in the standard weak interaction theory. It means that in this interaction the additive numbers cannot be conserved. These results are the consequence of specific character of the weak interaction: the right components of spinors do not participate in this interaction. The schemes of violation of the aromatic and lepton numbers were considered.

Appendix

Under the chiral transformation (12)

$$\Psi(x)' \to exp(i\alpha(x)\gamma_5)\Psi$$

the coefficient a_n of the following expansions:

$$\begin{array}{l} \Psi(x) = \sum_n a_n \phi_n \\ \bar{\Psi}(x) = \sum_n \phi_n^+ \bar{b}_n \end{array} \qquad (a.1)$$

$$d\mu = \prod_x [DA_\mu(x)] \prod_{m,n} d\bar{b}_m da_n,$$

(where $\hat{D}\phi(x) = \lambda_n \phi_n$, $\int \phi_n^+(x)\phi_m(x)d^4x = \delta_{n,m}$ and a_n, b_m^+ are the elements of the Grassman algebra)
are transformed as

$$a'_n = \sum_m \int \phi_n^+ exp(i\alpha(x)\gamma_5)\phi_m dx a_m = \sum_m c_{nm} a_m. \qquad (a.2)$$

Then

$$\prod_n da'_n = (detC_{k,l})^{-1} \prod_n da_n \qquad (a.3)$$

where

$$(detC_{k,l})^{-1} = det(\delta_{k,l} + i\int \alpha(x)\phi_k^+(x)\gamma_5\phi_l(x)dx)^{-1} =$$
$$= exp(-i\int \alpha(x)\sum_k \phi_k^+(x)\gamma_5\phi_k(x)dx).$$

The summation in the exponent of (12) is a bad-defined quantity and evaluating it by introduction a cutoff M ($\mid \lambda_k \mid \leq M$) we have

$$\begin{array}{c} \sum_k \phi_n^+(x)\gamma_5\phi_k(x) = \lim_{M\to\infty} \sum_k \phi_k^+ \gamma_5 exp\left[-\left(\frac{\lambda_k}{M}\right)^2\right]\phi_k(x) = \\ = \lim_{M\to\infty, y\to x} tr\gamma_5 exp\left[-\left(\frac{\hat{D}}{M}\right)^2\right]\delta(x-y) = \\ = \lim_{M\to\infty, y\to x} \int \frac{dx}{(2\pi)^4} tr\gamma_5 exp\left[-(D^\mu D_\mu + \frac{1}{4}[\gamma^\mu,\gamma^\nu]F_{\mu\nu})/M^2\right] e^{ik(x-y)} = \\ = \lim_{M\to\infty} \frac{1}{16} tr\gamma_5([\gamma^\mu,\gamma^\nu]F_{\mu\nu})^2 \frac{1}{2M^4}\int \frac{dk}{2\pi^4} exp^{(-\frac{k^2}{M^2})}. \end{array} \qquad (a.4)$$

After the integration one abtains

$$\sum_k \phi_k^+(x)\gamma_5\phi_k(x) = -\frac{1}{16\pi^2} tr^* F^{\mu\nu} F_{\mu\nu} \qquad (a.5)$$

The same result one obtain at b_n^+ transformation, and as a result one gets eq. (15), i.e.

$$d\mu' = d\mu e^{\left(i\int \alpha(x)(\frac{1}{8\pi^2})^* F^{\mu\nu} F_{\mu\nu}\right)}.$$

It is clear that in the case of the weak interactions, since $\phi_{Rk}^+(x) = \phi_{Rk} \equiv 0$ we have

$$\sum_k \phi_{Lk}^+(x)\gamma_5\phi_{Lk} \equiv 0 \qquad (a.6)$$

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